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HYDROPHYSICAL PROCESSES

Modeling Fast Ice Formation and Destruction in the Sea of Japan

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Abstract—The problems of parametric representation of the initial formation and the subsequent evolution of fast ice in freezing seas are discussed within the framework of the model of marine ice cover evolution. The mathematical form of this representation takes into account the processes of transformation of sea ice formations in open water areas in coastal regions into fast ice during its formation and inverse processes at the stage of its destruction. The parametric identification of the model was based on samples of long-term distributions of ice cover characteristics in the Sea of Japan, as well as the distributions of air temperature and wind speeds over sea water area. The model can be used to predict the state of fast ice in the ice cover of the Sea of Japan.

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INTRODUCTION

Fast ice is an immovable ice formation, which, along with open-sea ice forms the ice cover (IC) in the coastal zone. The need to study fast ice and to model its evolution is determined by the requirements of scientifically sound prediction, which is of importance for ensuring the safety of human life and the development of mineral resources on the shelf of freezing seas, as well as the construction of hydroengineering structures and navigation.

A specific feature of the suggested approach is the use of statistics for the formalization of the models developed. It is assumed that the assemblage of individual floes in a sea water area corresponds to an ensemble of interacting particles with different areas and thicknesses. This allows one to jointly take into account the thermal and wind external atmospheric impact on IC, the aggregation and destruction of floes and hummocking processes. In the coastal zones, the model accounts for the passage of ice of open-sea areas into fast ice during the formation of IC in autumn and the reverse processes during ice melting. The effect of the factors the combination of which under natural conditions causes the stabilization of IC thickness is also taken into account.

The field data are presented by V.V. Plotnikov and V.P. Tunegolovets (Pasific Oceanological Institute, FED, RAS). V.V. Plotnikov presented a sample of distributions of IC characteristics for the Sea of Japan over 1961–1989, and V.P. Tunegolovets provided a sample of temperature and wind speed over 1960–2001.

FAST ICE OF THE SEA OF JAPAN

IC of the Sea of Japan is a heterogeneous space and time system. Therefore, the study of the specific features of fast ice formation, mature state, and melting is possible only with a detailed analysis of the dynamics of fast ice characteristics in individual coastal zones of the water area. The IC of Amur Bay and the northern Tatar Strait are considered in this study. The choice of these areas is determined by their important strategic location and the distinct evolution stages. Moreover, Amur Bay is a sea region, which is relatively closed to the inflow of ice formations from outside. Therefore, it is of particular interest for the parameterization of the mechanism of distribution of IC state characteristics in individual regions. Figure 1 gives 10-day-averaged distributions of the area and thickness of IC in the water areas of Amur Bay (Figs. 1a, 1b) and the northern Tatar Strait (Fig. 1c, 1d), where the curves marked by squares characterize open-sea ice, and circles characterize fast ice. Hereinafter, we use the appropriate sample distributions averaged over a many-year observational period. Additionally, the areas of parts of the sea are expressed in terms of the units of the area of the open-sea region.

The primary forms appear in Amur Bay water area between the 33rd and 34th ten-day intervals in the year. Within these intervals, ~4% of the open-sea area is covered by ice with a first gradations of thickness (up to 0.15 m). Next, it grows rapidly, and by the beginning of the 2nd ten-day interval, 29% of the open-sea area is covered by ice. The destruction of IC begins in the 8th ten-day period. In the following two ten-day intervals, ice area drops from 30 to 7% of the open-sea area. Analysis of Fig. 1 shows the presence of some “competition” between the fast ice and open-sea ice areas: except for the periods of initial formation and the period of final destruction, there are no periods when these areas would simultaneously grow or drop. Moreover, an increase in the fast ice area is accompanied by a drop in the open-sea ice area, and the destruction of fast ice takes place against an increase in the open-sea

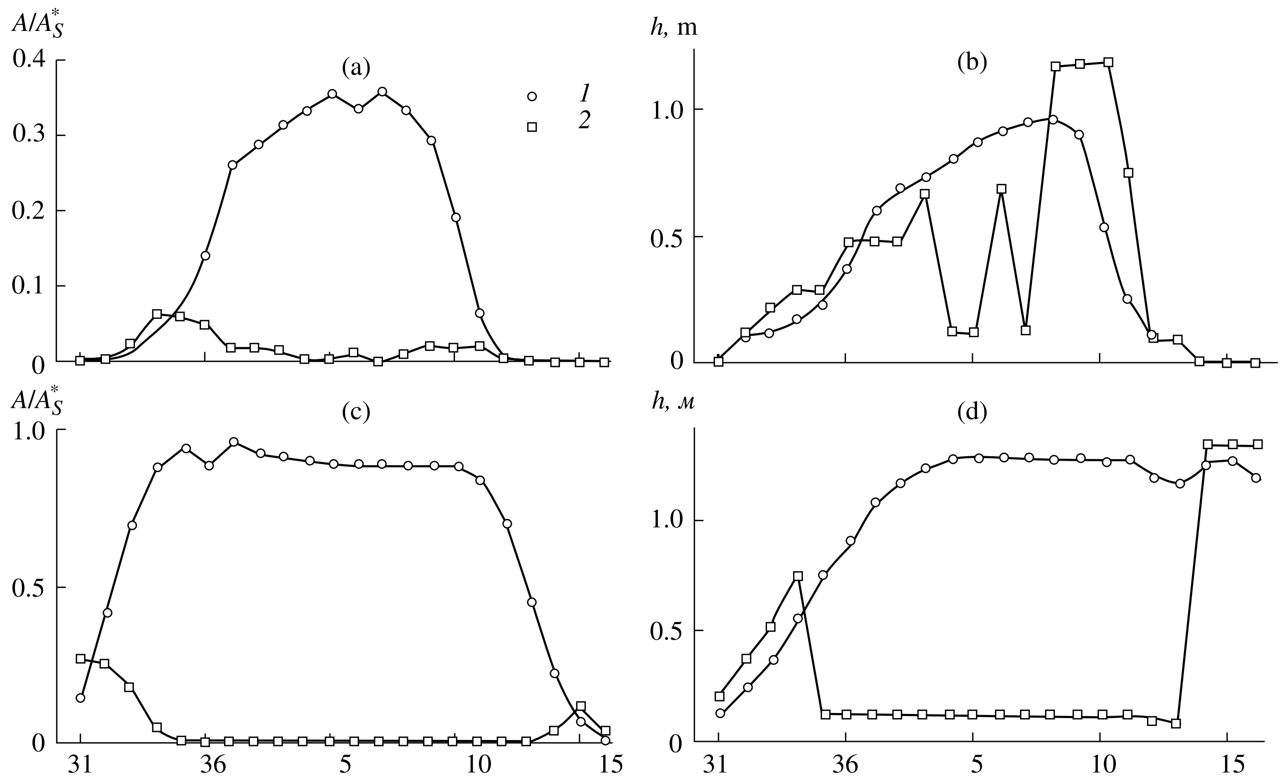

Fig. 1. Ten-day-averaged distributions of (a, b) the area A and thickness h of fast and open-sea ice in the water areas of Amur Bay and (c, d) the northern Tatar Strait. Here and in Fig. 2: (*1, 2*) fast and open-sea ice, respectively (the abscissa corresponds to ten-day period of the year; the areas are normalized by the open-sea area A_s^*).

area. This phenomenon is not surprising, because fragments of fast ice that form during its destruction pass into the category of open-sea ice. During the major portion of the evolution cycle of IC in the coastal region, the fast ice dominates over open-sea ice. The rate of ice cover formation and destruction in northern Tatar Strait is much greater. The relationships described above also hold for the fast ice and open-sea ice in this area.

This is also true for the distributions of mean ice cover thickness. The presence of thick ice at the final stage of IC evolution is due to the following. In this case we consider not the mean ice thickness in the regions under study (the areas and thicknesses are spread by a uniform layer over the water areas of the regions) but the values of the thickness of real ice formations averaged over the observational period of a specific ten-day period

$$\bar{h}_r(d) = \sum_{Y=1961}^{1989} h_{rY}(d)A_{rY}(d)/\sum_{Y=1961}^{1989} A_{rY}(d),$$

where r is the number of the region within the water area; Y is the observation year; d is the ten-day period within the year; $h_{rY}(d)$ and $A_{rY}(d)$ are the thickness and the area of IC in region r in year Y recorded in ten-day period d. Since the thermal destruction of CI area is much faster than the thermal decrease in ice thickness, thick ice with small area is present at the final stage of ice melting.

An interesting question is the relationship between the total fast ice area and the total area of the open-sea IC. Figure 2 presents ten-day averages of ice coverage distribution (expressed in the percentage of the total ice area in the sea region to the area of this region) and the volumes of ice in the water area of the Sea of Japan, where the curves marked by squares characterize the ice in open-sea areas and circles characterize fast ice. The fact that the volume of ice is considered instead of the mean thickness of IC is due to the fictitious character of the former value: when ice cover in the northern part of the sea is already mature, there may be no ice at all in the southern part of the sea. Thus, in the northern part of the sea, the duration of the evolution cycle varies from 18 to 22 ten-day periods, while that in the southern part varies from 10 to 14 ten-year periods. Note that the area-averaging of IC characteristics results in a uniform distribution of their values over the entire sea area. Therefore, the mean IC thickness of the sea is a fictitious rather than real characteristic. The volume of ice is a real value.

Calculations show that the fast ice area attains its maximum during the period of the largest value of ice coverage of the Sea of Japan. The curves were constructed taking into account the areas of individual

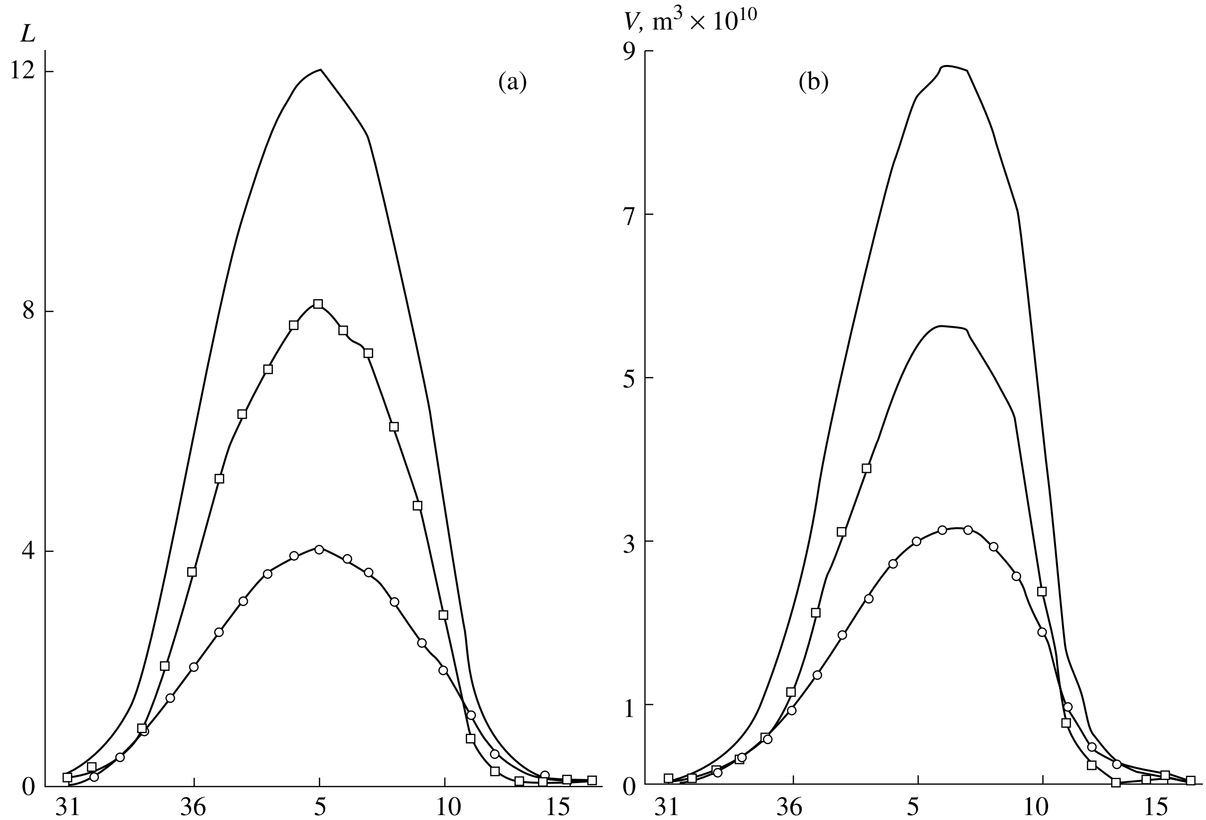

Fig. 2. Ten-day-averaged distributions of ice coverage and volume in the water area of the Sea of Japan (the full lines are the resulting distributions).

regions, which in accordance with the accepted practice of the examination of marine IC are expressed in terms of conventional units of area of the open-sea region. The synchronous character of variations in the fast ice area and ice coverage is also confirmed by the high value of their correlation coefficient, which is equal to 0.879 (the same coefficient for the total ice volumes is 0.981). This regularity is due to the fact that, because of the regional specifics of the ice regime of the Sea of Japan, the stage of the mature state of its fast ice coincides with the stage of the mature state of open-sea ice. Therefore, the state of fast ice largely determines the state of IC in the open-sea area and vice versa. This takes place, notwithstanding the essential distinctions between the areas of fast ice and open-sea ice. Such relationships also hold for some other characteristics of marine IC, in particular, the total volumes of ice.

The properties mentioned above can be described by the linear regression equations

$$\begin{aligned} L_S &= k_L^{(1)} + k_L^{(2)} L_F, \\ V_S &= k_V^{(1)} + k_V^{(2)} V_F \end{aligned} \tag{1}$$

where L_S, L_F are the ice coverage values for the open sea and fast ice; V_S, V_F are the respective total ice volumes; $k_L^{(1)}$, $k_L^{(2)}$, $k_V^{(1)}$, and $k_V^{(2)}$ are coefficients of regression lines, where the dependent variables are characteristics of open-sea IC, and the independent variables are fast ice characteristics. The coefficients of (1) are evaluated from the long-term averages of sample distributions: $k_L^{(1)} = -0.46791 \pm 0.31403$, $k_L^{(2)} = 2.08184 \pm 0.13634$, $k_V^{(1)} = -(2.77353 \pm 1.99746)\ 10^9$, $k_V^{(2)} = 1.80438 \pm 0.11339$. The values of the respective coefficients of determination of equation (1) equal to 0.773 and 0.795 suggest the close linear relationship between the measured values of dependent variables and those calculated by (1) (the coefficient of determination characterizes the percentage of the total variation of the dependent variable that is accounted for by regression). The confidence intervals of parameter values show their statistically significant difference from zero. Notwithstanding the large errors in their estimates $k_L^{(1)}$ and $k_V^{(1)}$, the negative values convincingly demonstrate that at the scale of the sea, the initial appearance of ice in the open sea is preceded by the appearance of fast ice. Indeed, the values of ice coverage and volumes in the open sea are positive only when the characteristics of fast ice exceed certain values.

A KINETIC EVOLUTION MODEL OF MARINE ICE COVER

The analysis of joint sample distributions of the areas of individual floes a and their thicknesses h [10] suggests their statistical independence. Therefore, the assemblage of individual sea ice formations (floes and hummocks) within each sea region can be ordered on the basis of the table $\Omega = \{(a, h)\colon 0 < a \le A^*, 0 < h \le H^*\}$ of sizes a and h. Here, A^* is the total area of a region (the areas of coastal regions are, as a rule, less than that of open regions A_S^*); H^* is the maximum IC thickness (estimated from long-term observations); a_1 is the minimal floe area (the minimal area of an ice formation that discriminates it from an accumulation of ice nucleation centers). According to this assumption, the result of contact interaction between individual floes is an element of this table. The following assumptions are also made: mass ice drift takes place; the thermal regime of the atmosphere determines the successive change of ice floe dimensions a and h to other values; the thermal regime determines the primary and subsequent formation of floes and their melting; during the formation of ice cover in coastal regions, the open-sea ice passes into fast ice; while an inverse process takes place during ice melting; hummocking causes a decrease in thin ice area and an increase in the area of thick ice; the processes of aggregation and destruction of individual floes take place. These assumptions are justified by the analysis of observations and the accepted space and time scales.

In the construction of the kinetic model, the dynamic variable is the distribution density in terms of (a, h) of the number of floes in the open areas of the region $n \equiv n(x, t, a, h)$ and the fast ice floes $n^{(F)} \equiv n^{(F)}(x, t, a, h)$ for open-sea areas $n^{(F)} \equiv 0$), where $x = (x_1, x_2)$ are space coordinates. Under the above assumptions, the space and time dynamics of floes in the open part of the regions is determined by the balance relationship

$$\begin{aligned}\partial n/\partial t + \partial(u_i n)/\partial x_i + \partial(\dot{a}n)/\partial a + \partial(\dot{h}n)/\partial h \\ = f_{ah} + b_{ah} + \Psi_{ah} + Q_{ah} + R_{ah},\end{aligned} \tag{2}$$

where $u = (u_1, u_2)$ is ice drift velocity; $\dot{a} \equiv da/dt$, $\dot{h} \equiv dh/dt$ are variations in a and h determined by the thermal regime; $f_{ah} \equiv f_{ah}(x, t, a, h)$ is the rate of appearance (disappearance) of floes with the area of a and the thickness of h directly from seawater (or into seawater), bypassing the stages of their transformation from some areas and thicknesses into others; $b_{ah} \equiv b_{ah}(x, t, a, h)$ is the variation of n in the open parts of the water area of the coastal region per unit time, which at the stage of IC formation in autumn is determined by the passage of open-sea floes into fast ice and an inverse passage of fast ice fragments into open-sea floes during its spring melting and destruction ($b_{ah} \ge 0$ only in coastal regions); $\Psi_{ah} \equiv \Psi_{ah}(x, t, a, h)$ characterizes changes in n because of ice hummocking; $Q_{ah} \equiv Q_{\mathrm{ah}}(x, t, a, h)$, $R_{ah} \equiv R_{ah}(x, t, a, h)$ characterize the dynamics of ice aggregation and destruction per unit time. The deviations of the drift of individual floes do not exceed 5% of their averaged motion [4]. Therefore, equation (2) is written with n assumed to be independent of velocity scatter.

Formulas for $\dot{a}$ are written with the use of the concepts accepted in resource–consumer systems [5]. Floes are assumed to share their common "resource" of ice-free water areas $A^* - A_I \equiv A^* - \int_0^{H^*}\int_0^{A^*} a(n + n^{(F)})\,da\,dh,$.

$$\begin{aligned}\dot{a} &= (T^* - T)[\alpha_a^{(A)}\Theta(T^* - T) \\ &+ \alpha_a^{(S)}\Theta(T - T^*)](A^* - A_I)a,\end{aligned} \tag{3}$$

where $\alpha_a^{(A)}$, $\alpha_a^{(S)}$ re nonnegative coefficients (the subscript denotes the evolution stage: (A) autumn and (S) spring; $\Theta(z)$ is Heaviside function; T^* is air temperature at which primary floes will form and the initial ice melting takes place. Statistical analysis of temperature sample distributions shows a statistically significant coincidence between air temperature at the initial formation of IC and its initial melting [7]. The difference between T^* and the temperature of initial formation of fast ice T_F^*. is obvious. Indeed, waters in the coastal zones where fast ice is located are relatively shallow and fresh because of their freshening by river waters and industrial wastes. Therefore, the formation of ice in open-sea areas begins at lower air temperature than in the fast-ice zones. Ice melting in open areas begins at lower temperature: in this period, the combined effect of solar radiation and warm sea currents manifests itself [11].

The "resource" for thickness is the seawater layer $H^* - h$ available for it [9]. With allowance made for the formation of melt water within the IC mass (water of puddles and water that forms during ice melting), the approximation of $\dot{h}$ takes the form

$$\begin{cases}\dot{h}(t, T^*) = (T^* - T)\{[\alpha_h^{(A)}\Theta(T^* - T) \\ + \alpha_{hh}^{(S)}\Theta(T - T^*)](H^* - h - h_W)h + \alpha_{Wh}h_W\}, \\ \dot{h}_W(t, T^*) = (T - T^*)(\alpha_{hW}h - \beta_W h_W)\Theta(T - T^*),\end{cases} \tag{4}$$

where $\alpha_h^{(A)}$, $\alpha_{hh}^{(S)}$, α_{Wh}, α_{hW}, β_W are nonnegative coefficients; h_W is the melt water layer in IC mass. Formula (4) was derived under the assumption that the melt water forms only at the stage of IC melting. Since the albedo of water is far less than that of ice (that is why the melt water is warmer than ice), the volumes of melt water, against the background of air warming typical of spring time, cause additional melting of IC (term $\alpha_{Wh}h_W$ in the equation for $\dot{h}$). It is also supposed that $\dot{h}_W \sim h$and melt water discharge through IM mass takes place (term $\beta_{Wh}h_W$).

Formula for f_{ah} is derived under the assumption that, during the formation of ice cover, $I \equiv \int_0^{H^*}\int_0^{A^*} f_{ah} da dh$ is the total number of floes that appear per unit time, and during ice melting, it is the number of melted floes. It is appropriate to assume that in the first case I is proportional to the resource $A^* - A_I$ available for ice, while in the second case, it is proportional to the current ice area A_I. In both cases, the rate of the processes is determined by the difference $T^* - T$. It appears reasonable to write f_{ah} as

$$f_{ah} = (T^* - T)[(A^* - A_I) f_a^{(A)}(a) f_h^{(A)}(h)\Theta(T^* - T) + A_h f_a^{(S)}(a) f_h^{(S)}(h)\Theta(T - T^*)], \quad (5)$$

where $f_a^{(A)}(a)$, $f_h^{(A)}(h)$, $f_a^{(S)}(a)$, and $f_h^{(S)}(h)$ are rapidly decreasing functions of their arguments; $A_h(x, t, h) \equiv \int_0^{A^*} an(x, t, a, h) da$ is the total area of IC with a thickness of h in the water area of the region. Formerly it was assumed that only floes with initial thickness gradation appear/disappear per unit time [7, 8]. The reason to consider the appearance/disappearance of floes with arbitrary size per unit time is the dependence of f_{ah} on the length of the time step of the model and the temperature difference $T^* - T$, since, as the model time step increases, the thickness of the primary floes can exceed the thickness of their initial gradation.

The parametric representation of the dynamics of change of open-sea ice into fast ice at the stage of IC formation and the inverse change during ice melting b_{ah} is based on modified relationships taken from [7–10]

$$\begin{aligned} b_{ah}(T, a, h) &= -b_a^{(A)}(T, u) n(a, h)\Theta(T^* - T) + b_a^{(S)}(T, h, W) n^{(F)}(a, h)(A^* - A_I)\Theta(T - T_F^*), \\ b_a^{(A)}(T, u) &= [b_a^{(A,0)} + (T^* - T) b_a^{(A,T)} + b_a^{(A,u)} \vec{n} \circ u], \\ b_a^{(S)}(T, h, W) &= [b_a^{(S,0)} + (T - T_F^*) b_a^{(S,T)} - b_a^{(S,h)} h - b_a^{(S,W)} \vec{n} \circ W], \end{aligned} \quad (6)$$

where $b_a^{(A,0)}$, $b_a^{(A,T)}$, $b_a^{(A,u)}$, $b_a^{(S,0)}$, $b_a^{(S,T)}$, $b_a^{(S,h)}$, $b_a^{(S,W)}$, are nonnegative coefficients; $W = (W_1, W_2)$ is wind speed; $\vec{n}$ is normal to the shoreline; $-b_a^{(A)}(T, u)n(a, h)$ characterizes the transformation of open-water areas of coastal zone into fast ice during the formation of ice cover, while $b_a^{(S)}(T, h, W)n^{(F)}(a, h)(A^* - A_1)$ accounts for the transformation of fast ice into floes of open-water areas during ice cover melting and thermal destruction. Factor $A^* - A_1$ in the latter expression accounts for the fact that the destruction of fast ice and the joining of its fragments to open-sea ice require open water. The more is the share of open water, the more active is the process. Formula (6) also accounts for the fact that the rate of fast ice destruction is a decreasing function of ice thickness h. Only linear terms of approximations $b_a^{(A)}(T, u)$ and $b_a^{(S)}(T, h, W)$ are taken into account in (6).

When writing the representation of Ψ_{ah}, we supposed that the assemblage of sea ice formations $\Omega = \{(a, h) : 0 < a \le A^*, 0 < h \le H^*\}$ is a closed system with respect to hummocking processes (the cases of some floes passing under others are regarded as part of hummocking process). In other words, the result of hummocking of floes (elements of Ω) is also a floe with certain dimensions (an element of Ω). Observations and simple reasoning show that hummocking reduces the area of thin ice and increases the area of thick ice; some floes pass into the subsequent gradations; the total volume of ice remains constant (at the space and time scale accepted in this study, the losses of ice volume because of its compression during the contact can be neglected). Therefore, it appears natural to write Ψ_{ah} from (2) (note that this term characterizes the changes in floe areas because of hummocking) in the form

$$\begin{aligned} \Psi_{ah} &= -[\nu_1^{(A)}\Theta(T^* - T) + \nu_1^{(S)}\Theta(T - T^*)]\{(H^* - h)n - \chi \int_0^{h-\Delta h} (H^* - z) A_z w_{ah}(h, z) dz\}, \\ \chi &= \int_0^{H^*} (H^* - h) h A_h dh / \int_0^{H^*} h \times \int_0^{h-\Delta h} (H^* - z) A_z w_{ah}(h, z) dz dh, \end{aligned} \quad (7)$$

where $\nu_1^{(A)}$, $\nu_1^{(S)}$ are nonnegative coefficients; Δh is the specified h step; $w_{ah}(h, z) \ge 0$ is a function used to formulate the redistribution of the area of ice with a thickness of h into floes with a thickness of z ($h < z \le H^*$). When writing Ψ_{ah}, we assume that the decrease in n during hummocking per unit time is proportional to its current value; the greater ice thickness, i.e., the less $H^* - h$, the less its hummocking (this results in that the area of the thickest ice does not decrease during hummocking); the increment in n is due to the passage into this gradation of ice areas from previous thickness gradations (the second term in curly brackets). Clearly, the greater the current total area A_h of ice with a thickness of h, the greater the rate of its hummocking. That is why, the expression under integral sign contains floe areas, rather than the respective densities. According to the formula for χ, the volumes of ice do not change during hummocking (this condition is certainly true, if the compressibility of ice can be neglected). We propose the following scheme for the representation of $w_{ah}(h, z)$: during hummocking, the largest increment in the area is recorded only for the thickness gradation next to the initial one, and next the increments in areas decrease as functions of $z - h$. Now the approximation of $w_{ah}(h, z)$ takes the form

$$w(h, z) = \mathrm{const} \cdot \exp\{-[\nu_2^{(A)}\Theta(T^* - T) + \nu_2^{(S)}\Theta(T - T^*)](z - h)\}, \tag{8}$$

where $\nu_2^{(A)}$, $\nu_2^{(S)}$ are nonnegative coefficients. The properties of model (7) directly follow from its parametric representation: the maximum losses of n during hummocking take place for the thinnest floes. Additionally, if only ice of the last thickness gradation (the thickest floes) are present in a water area, the second and third terms in the square brackets of the first formula vanish and thus there is no ice hummocking.

Air temperature drop causes freezing up of small ice pieces. Therefore, aggregation is significant only at the initial stage of ice cover formation, when the assemblage of floes in the water area is so disperse that only their pair collisions can be taken into account (higher order collisions can be neglected). At this stage, it appears admissible to write $Q_{ah} \equiv Q(a, h)$ with the use of appropriate modification from [8]. If the result of floe destruction is assumed to have the thickness of the initial floe, the form of representation of R_{ah} from (2) is determined by relationships used in [2, 7, 8, 10].

The initial distribution is assumed to be zero. The boundary conditions for (2) naturally follow from (3)–(8) and reflect the absence of the respective fluxes at the boundaries of the variation ranges of a and h,

THE DISTRIBUTION OF ICE COVER THICKNESS

The model equations are obtained by multiplying (2) by a and integration of the obtained result with respect to this variable considering (3)–(8)

$$\begin{cases} \partial A_h/\partial t + \partial u_i A_h/\partial x_i + \partial \dot{h} A_h/\partial h \\ = (T^* - T) f_A(A_h, T^*) + b_h + \Psi_A(A_h, T^*), \\ \partial A_h^{(F)}/\partial t + \partial \dot{h} A_h^{(F)}/\partial h = (T_F^* - T) f_A(A_h^{(F)}, T_F^*) \\ - b_h + \Psi_A(A_h^{(F)}, T_F^*), \end{cases} \tag{9}$$

where $A^{(W)}(x, t) \equiv A^* - \int_0^{H^*} [A_h(x, t, h) + A_h^{(F)}(x, t, h)]dh$ is the area of the open water within the water area of the coastal region; $b_h = \{-b_a^{(A)}(T, u)A_h\Theta(T^* - T) + b_a^{(S)}(T, h, W)A_h^{(F)}A^{(W)}\Theta(T - T_F^*)\}$; $f_A(A, \tilde{T}) = [\alpha_a^{(A)}A + f_{A,h}(h, \beta_1^{(A)}, \beta_2^{(A)})]A^{(W)}\Theta(\tilde{T} - T) + [\alpha_a^{(S)}A^{(W)} + f_{A,h}(h, \beta_1^{(S)}, \beta_2^{(S)})]A\Theta(T - \tilde{T})$; $\Psi_A(A_h, \tilde{T}) = -[\nu^{(A)}\Theta(\tilde{T} - T) + \nu^{(S)}\Theta(T - \tilde{T})][(H^* - h)A_h - -\chi\int_0^{h - \Delta h}(H^* - z)A_z w_{ah}(h, z)dz]$; $b_a^{(A)}(T, u)$ and $b_a^{(S)}(T, h, W)$ are determined by relationships (6), χ ~ is determined by (7), and $w_{ah}(h, z)$ ~ by (8); $f_{A,h}(h, \beta_1, \beta_2) = \beta_1\exp(-\beta_2 h)$; $\beta_1 = \int_0^{A^*} f_a(a, h)da$. The distribution of fast ice thickness $A_h^{(F)}(x, t, h) \equiv \int_0^{A^*} an^{(F)}(x, t, a, h)da$ is written with the following considerations taken into account. The fast ice can be formally represented by density $n^{(F)}(x, t, a, h)$, where the number of floes in each gradation of a and h is equal to 0 or 1. Now (2) also determines the evolution of the distribution density $n^{(F)}(x, t, a, h)$ of the floes in the fast ice, if we set the ice drift velocity in its left-hand part equal to zero.

Formula $J(h) \equiv \int_0^{A^*} a f_{ah}(a, h)da$ for the total area of the floes with a thickness of h that form/disappear per unit time is based on the following assumptions. First, the rapidly decreasing function $f_h(h)$ in (5) is represented by the function $f_h\{h, \beta_1, \beta_2) = \beta_1\exp(-\beta_2 h$, where β_1, β_2 are nonnegative coefficients. Second, when writing the formula for $J(h)$, we suppose that at the stage of ice cover formation, the total area of the newly formed floes with a thickness of h is proportional to the open-water area, while at the stage of melting, it is taken proportional to the total area corresponding to this thickness. Now, if $\beta_1^{(A)}$, $\beta_2^{(A)}$, $\beta_1^{(S)}$, and $\beta_2^{(S)}$ are the values of the respective coefficients for the stages of formation and melting, we have

$$J(h) \equiv (T^* - T)[f_h(h, \beta_1^{(A)}, \beta_2^{(A)})A^{(W)}\Theta(T^* - T) + f_h(h, \beta_1^{(S)}, \beta_2^{(S)})A\Theta(T - T^*)].$$

When writing (9), we take into account the fact that in terms of individual thicknesses, the redistribution of ice (aggregation and destruction) does not change their total area $\int_0^{A^*} aQ_{ah}da = 0$ and $\int_0^{A^*} aR_{ah}da = 0$. The initial and boundary conditions for A_h and $A_h^{(F)}$ follow from (3)–(7)

$$A_h(x, t_0, h) = A_h^{(F)}(x, t_0, h) = 0$$
$$\text{и} \quad \dot{h}A_h\big|_{h = 0, H^*} = \dot{h}A_h^{(F)}\big|_{h = 0, H^*} = 0. \tag{10}$$

Qualitative analysis of equations (9) shows that when $T^* - T$ is positive, the system has one stationary state, which corresponds to the mature state of the cover at its positive value (the final stage of ice cover formation, when only thick ice is present in the water area), while when this difference is negative, this stationary state corresponds to the absence of ice in the water area (the final stage of IC evolution cycle).

The integration of system (9) with respect to h yields equations for the total area of IC in the water area of the region. If the IC of the coastal area is represented only by fast ice (relatively closed regions of the sea water area) and hummocking is not taken into account, the total area of fast ice in the water area of the region $A^{(F)}(t)$ is determined by the equation $\dot{A}^{(F)} = (T_F^* - T)\{(\alpha_a^{(A)} A^{(F)} + \alpha_A^{(A)})(A^* - A^{(F)})\Theta(T_F^* - T) + [\alpha_a^{(S)}(A^* - A^{(F)}) + \alpha_A^{(S)}]A^{(F)}(T - T_F^*)\}$, where $\alpha_A^{(A)} \approx \beta_1^{(A)}/\beta_2^{(A)}$ and $\alpha_a^{(S)} \approx \beta_1^{(S)}/\beta_2^{(S)}$.. This relationship is derived under the assumption that the initial gradation of the thickness is so small that $\exp(-\beta_2^{(A)} h_1) \approx 1$ and $\exp(-\beta_2^{(S)} h_1) \approx 1$; the values of $\beta_2^{(A)} H^*$ and $\beta_2^{(S)} H^*$ are such that the exponents with their negative values can be neglected. If the assumptions made during the derivation of this equation are valid (the fast ice accounts for the major portion of the IC in the coastal region), the character of variations in the fast ice area is determined by the temperature regime and the water area A^*

$$A^{(F)}(t) = A^{(A,F)}(t)\Theta(T_F^* - T) + A^{(S,F)}(t)\Theta(T - T_F^*),$$

$$A^{(A,F)}(t) = A^*\left\{1 - \frac{\alpha_A^{(A)} + \alpha_a^{(A)}}{\alpha_A^{(A)}\exp[(\alpha_a^{(A)}A^* + \alpha_A^{(A)})(T_F^* - \bar{T}^{(A)})(t - t_0^{(A)})] + \alpha_a^{(A)}}\right\}, \tag{11}$$

$$A^{(S,F)}(t) = \frac{\alpha_a^{(S)}A^* + \alpha_A^{(S)}}{\alpha_a^{(S)}}\left\{1 - \frac{1}{\alpha_a^{(S)}C_A^{(S)}\exp[(\alpha_a^{(S)}A^* + \alpha_A^{(S)})(T_F^* - \bar{T}^{(S)})(t - t_0^{(S)})] + 1}\right\},$$

where $t_0^{(A)}$, $t_0^{(S)}$ are the time moments corresponding to the initial stage of IC formation and the initial stage of its melting; $\bar{T}^{(A)}(t) = \frac{1}{t - t_0^{(A)}}\int_{t_0^{(A)}}^{t} T dt$ and $\bar{T}^{(S)}(t) = \frac{1}{t - t_0^{(S)}}\int_{t_0^{(S)}}^{t} T dt$ are the mean air temperatures from the moment of initial formation/melting of IC to the corresponding current time moment. In the first case, $A^{(F)}(t_0^{(A)}) = 0$, while in the second case, $A^{(F)}(t_0^{(S)})$ is the fast ice area at the initial moment of its melting stage and $C_A^{(S)} = A^{(F)}(t_0^{(S)})/[\alpha_a^{(S)}A^* + \alpha_a^{(S)} - \alpha_a^{(S)}A^{(F)}(t_0^{(S)})]$.

Analysis of the evolution of the total fast ice area at the stage of its formation shows the following. The time moment $t = t_A^{(A)}$ of the inflection of curve $A^{(A,F)}(t)$ is determined by the solution of equation $\dot{A}^{(A,F)} = 0$, i.e.,

$$-\dot{T}(t)/[T_F^* - T(t)]^2 + \alpha_a^{(A)}A^* - \alpha_A^{(A)} - 2\alpha_a^{(A)}A^{(A,F)}(t) = 0,$$

where $t_0^{(A)} < t < t_0^{(S)}$. Under the conditions that are natural for the formation of fast ice, the decrease in temperature $-\dot{T}(t)$ is nonnegative. Now the curve of the fast ice area has an inflection point, if the following conditions are met: $A^{(A,F)}(t_A^{(A)}) > 0$ or $\alpha_A^{(A)} < \alpha_a^{(A)}A^* - \dot{T}/(T_F^* - T)^2$; $A^{(A,F)}(t_A^{(A)}) < A^*$ or $-\dot{T}/(T_F^* - T)^2 < \alpha_a^{(A)}A^* + \alpha_A^{(A)}$. To exemplify the interpretation of the inequalities, let us consider the case $-\dot{T}(t_A^{(A)}) = 0$. Now there will be no inflection point on the curve if, at the same air temperature $T < T_F^*$, the water area of a small water body is being rapidly covered by ice film. Even when the first inequality is valid, the curve of the fast ice area may have no inflection point: this requires a certain regime of air cooling (the second inequality), for example, its abrupt cooling. For the stage of fast ice melting, the value $-\dot{T}(t)$ is negative. Therefore, the second inequality is valid at any nonnegative parameters, and the first inequality holds only at certain thermal regime of air temperature rise.

In this case, the time step is taken equal to 1 day, which is due to the discreteness of the sample distributions of temperature and wind speed. Observations and the analysis of the order of magnitude of the terms of the appropriate equations suggest that at this time step, the drift velocity of ice cover (rather than individual floes) has a quasi-stationary character and is determined by simple relationships [6]. Figure 3 gives the distributions of the thickness of ice $A_h^{(F)}$ and A_h of different thickness values in the water areas of Amur Bay (Figs. 3a–3c) and the northern Tatar Strait (Figs. 3b–3d). To facilitate their analysis, the distributions of IC in open-sea areas are placed under the distribution of fast ice. Analysis of individual cases shows that the areas of IC of individual zones increase at the stage of spring ice destruction and melting, which is a consequence of the transfer of fast ice into these zones.

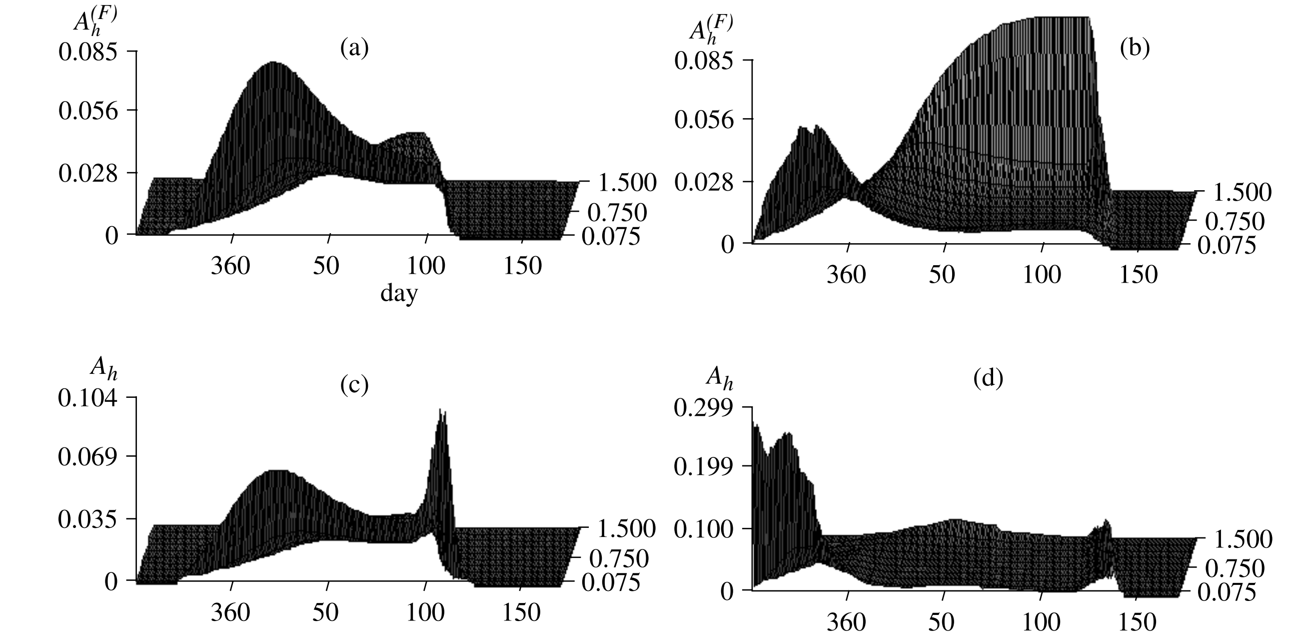

Fig. 3. The distribution of IC thickness in (a, c) Amur Bay water area and (b, d) northern Tatar Strait; (a, b) fast ice, (c, d) open-sea areas.

To elucidate the causes of open-sea ice transformation into fast ice and the inverse transformation in these regions let us consider the distribution of individual components in (6) given in Fig. 4. The comparison of cases (a) and (b) reveals differences between fast-ice evolution in the regions considered. Thus, in the first case, the thermal (the second term in $b_a^{(A)}(T, u)$) and the wind (the third term in $b_a^{(A)}(T, u)$) regimes of open parts of the water area have no significant effect on fast-ice formation. At the stage of IC melting in this region, the thermal impact is the key factor governing the destruction of fast ice, as it can be seen from the excess of the thermal component $(T^* - T)b_a^{(A,T)}\Theta(T^* - T)\int_0^{H^*} A_h\,dh$ + $(T - T_F^*)b_a^{(\dot{S},T)}\Theta(T_F^* - T)A^{(W)}\int_0^{H^*} A_h\,dh$ over the wind component $b_a^{(A,u)}\vec{n}\circ u\,\Theta(T^* - T)\int_0^{H^*} A_h\,dh$ $+\, b_a^{(\dot{S},W)}\vec{n}\circ W\,\Theta(T_F^* - T)A^{(W)}\int_0^{H^*} A_h^{(F)}\,dh$. At the stage of fast ice formation in the northern Tatar Strait, squeezing drift of ice takes place at $\vec{n}\circ u < 0$. In this case, the difference between the thermal and wind components of the right-hand part of (6) is insignificant. At the stage of melting, the thermal component plays certain role in the process of fast-ice destruction. This situation is opposite to the pattern of fast-ice destruction in the Gulf of Finland, where, according to the opinion of Z.M. Gudkovich and S.V. Klyachkin, the wind component is the key factor determining the fracturing of this fast ice [3]. However, direct observations show that the fracturing of fast ice in the Sea of Japan takes place even without the offshore wind: in windless and clear-sky days, the collapse and destruction of the front edge of fast ice takes place. Therefore, the mechanism (6) appears to better describe the destruction of fast ice.

PARAMETERIC IDENTIFICATION

The parameteric identification of model (9)–(10) is implemented by using the sample of areas of ice with different thickness averaged over the observational period. Element $A_{r,d,g}^{(D)}$ of the sample determines the area of the gth gradation of ice thickness in the dth ten-day period in the rth region (6 uneven gradations of thickness are used). Element $A_{r,d,g}^{(D)}$ corresponds to $A_{r,d,g}^{(M)}(p) = 0.1\sum_{t=10d-9}^{10d}\sum_{j\in J(g)}[A(x_r, t, h_j, p) + A^{(F)}(x_r, t, h_j, p)]$, where the set of parameters is determined by $p = (\alpha_h^{(A)}, \alpha_h^{(S)}, \alpha_{hh}^{(S)}, \alpha_{Wh}, \alpha_{hW}, \beta_W, \alpha_a^{(A)}A_S^*, \alpha_a^{(S)}A_S^*, \nu_1^{(A)}, \nu_2^{(A)}, \nu_1^{(S)}, \nu_2^{(S)}, \beta_1^{(A)}, \beta_2^{(A)}, \beta_1^{(S)}, \beta_2^{(A)}, b_a^{(A,0)}, b_a^{(A,T)}, b_a^{(A,u)}, b_a^{(S,0)}, b_a^{(S,T)}, b_a^{(S,h)}, b_a^{(S,W)},)$ and includes $k = 23$ elements and $\Delta_h = 0.15$ m; $J(g)$ are the numbers of intervals of uniform division $(0, H^*]$, which cover the gth interval of thickness gradations; $r = 1 \div 114$ varies from 1 to 114, and $g = 1 \div 6$ varies from 1 to 6. The estimates of required parameters are sought for by minimizing $\Phi(p)$

$$\Phi(p) = e^T(p)e(p), \tag{12}$$

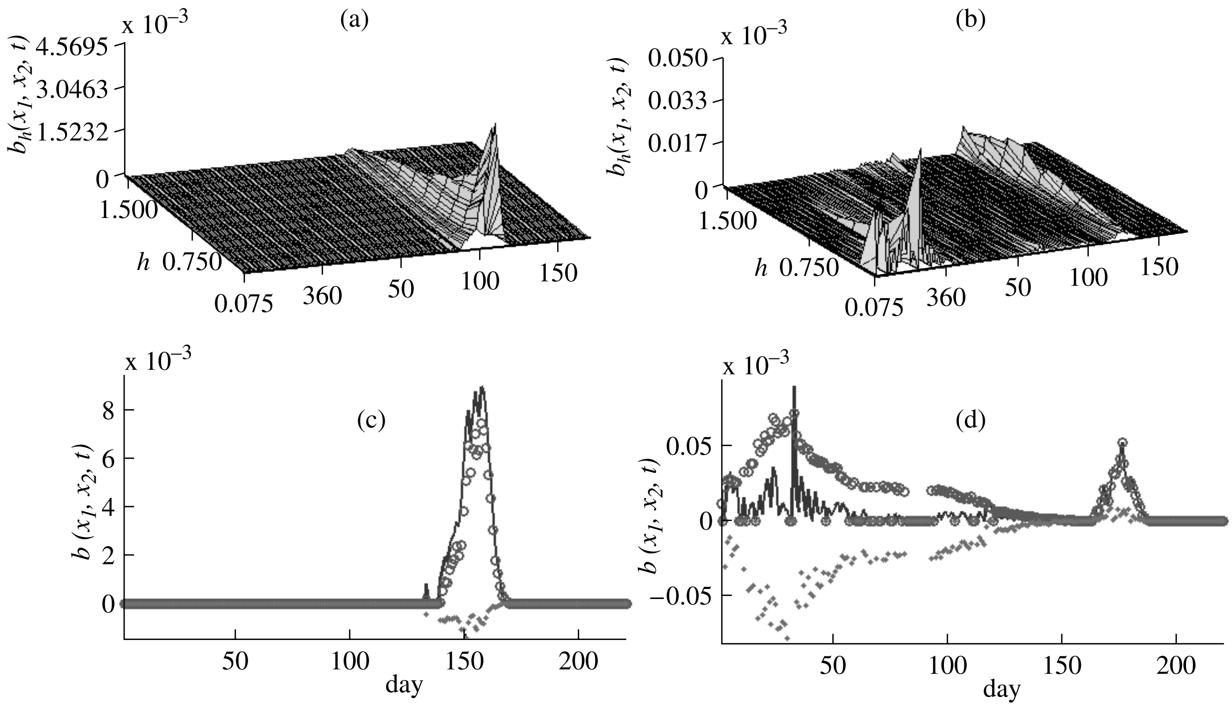

Fig. 4. Distributions of $b_h\,(x_1, x_2, t) = \int_0^{H^*} b_h\,(x_1, x_2, t)dh$ for (a, c) IC of Amur Bay and (b, d) northern Tatar Strait; circles and dots denote the distributions of the thermal and wind components of the process of fast ice formation–destruction in these regions (the full line is the resultant of the process).

where $e_\mu(p) = A^{(D)}_{rdg} - f_\mu(p)$ are the residuals of the assessment of model parameters; $\mu = 6\{\sum_{r'=1}^{r-1}[d_1\,(r') - d_0(r')] + d - d_0(r) + r - 1\} + g$; $d_0(r)$ and $d_1(r)$ are the initial and final ten-day periods of the evolution in the rth region; $f_\mu(p) = 0.1\sum_{d'=1}^{10}\sum_{j\in J_g} A\,(x_r, 10(d-1) + d', h_j, p)$.The formula for $f_\mu(p)$ was written taking into account that $A^{(D)}_{rdg}$ and $\sum_{j\in J_g} A\,(x_r, 10(d-1) + d', h_j, p)$ have different time scales: the former characterizes the ten-day value of the area, and the latter is its daily value. The initial approximation for the solution of the problem $\min ? \div \min_p ? \Phi(p)$ is determined from [7–10]. The solution of this problem yields the following estimates: $T^* = -7.6 \pm 0.8$; $T^*_F = -6.3 \pm 0.7$; $\alpha^{(A)}_h = (3.25213 \pm 1.41234)10^{-3}$; $\alpha^{(S)}_h = (2.00313 \pm 0.87690)10^{-3}$; $\alpha^{(S)}_{hh} = (6.75000 \pm 3.54372)10^{-3}$; $\alpha_{hW} = (2.53844 \pm 2.40695)10^{-3}$; $\alpha_{Wh} = 2.81969 \pm 0.68741$; $\beta_W = 4.74062 \pm 0.54146$; $\beta^{(A)}_1 = (7.06250 \pm 2.23817)10^{-4}$; $\beta^{(S)}_1 = (6.75114 \pm 0.86412)10^{-3}$; $\beta^{(A)}_2 = \beta^{(A)}_1 = 21.09382 \pm 2.41643$; $\alpha^{(A)}_a A^*_S = (2.68750 \pm 0.86265)10^{-2}$;$\alpha^{(S)}_a A^*_S = (1.93750 \pm 0.59215)10^{-2}$; $\nu^{(A)}_2 = 15.71884 \pm 4.38921$; $b^{(A,0)}_a = (5.53125 \pm 1.24344)10^{-2}$; $b^{(A,T)}_a = (1.49062 \pm 0.63352)10^{-2}$; $b^{(A,u)}_a = (5.94063 \pm 1.93126)10^{-2}$; $b^{(S,0)}_a = (5.53125 \pm 1.47089)10^{-2}$; $b^{(S,T)}_a = (7.75108 \pm 2.09674)10^{-2}$; $\nu^{(S)}_2 = 15.71884 \pm 4.38921$; $b^{(S,h)}_a = (3.74941 \pm 1.20877)10^{-2}$; $b^{(S,W)}_a = (1.75017 \pm 0.93516)10^{-2}$; $\nu^{(A)}_1 = (2.04688 \pm 0.64427)10^{-2}$; $\nu^{(S)}_1 = (7.05313 \pm 1.85319)10^{-3}$.

The values of the obtained parameter estimates are in agreement with the qualitative model of the character and the sequence of the stages of ice cover evolution in the Sea of Japan. The fact that $\beta^{(A)}_1 < \beta^{(S)}_1$ and $\nu^{(A)}_1 > \nu^{(S)}_1$ reflects the following situation. The penetration of short-wave radiation into ice results in that ice melting takes place not only on the upper surface (in the absence of clearings, the rate of melting on the upper surface can be 5–6 times greater than on the lower surface), but also on the sides, bottom, and within the ice mass (internal melting) [1]. Therefore, the melting rate of ice cover is greater than its formation rate even in the case when the distribution of the atmospheric temperature is symmetrical and $T^* - T$ for the autumn formation and $T - T^*$ for the spring destruction are equal to one another. Thus, it is known that, the atmospheric temperature being the same, the rate of ice melting in

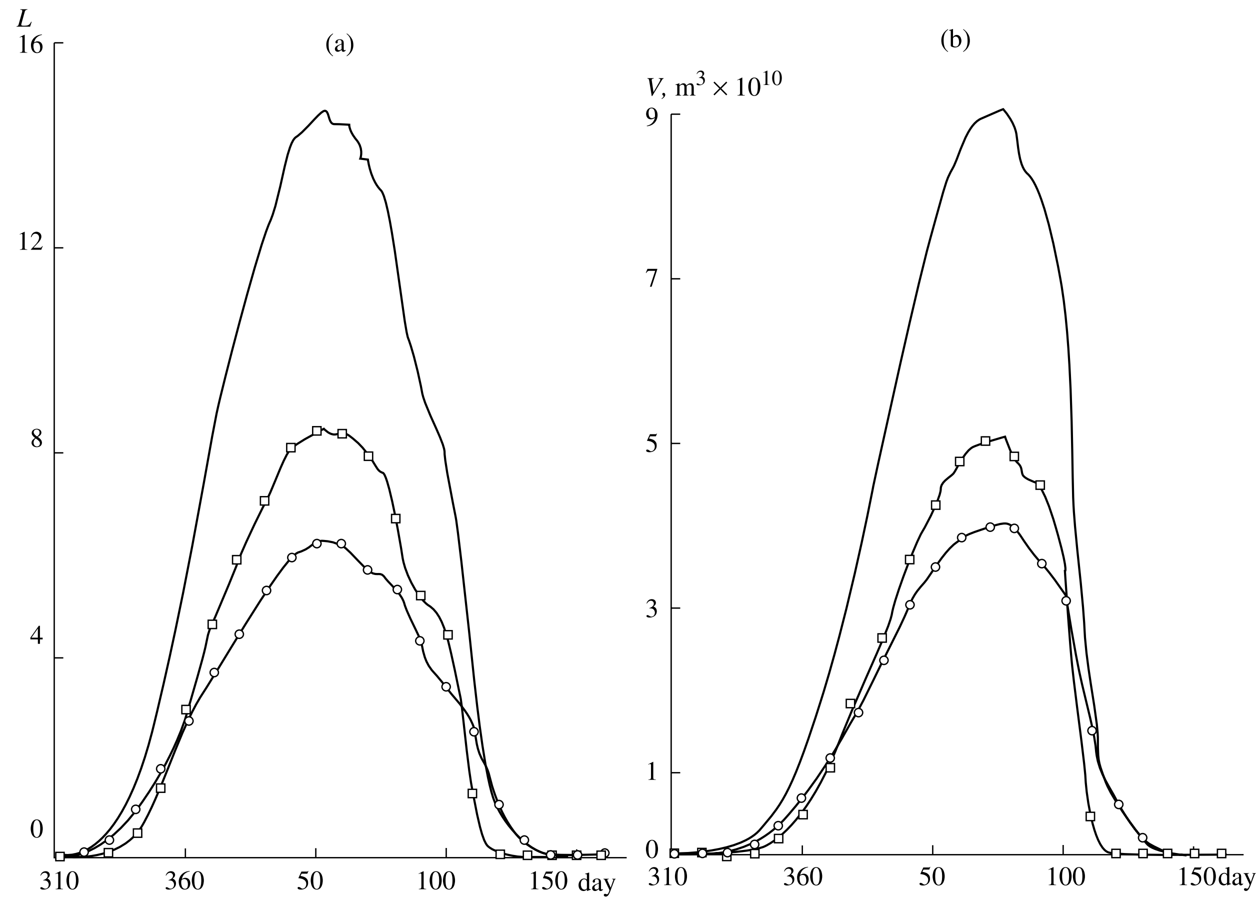

Fig. 5. The distribution of ice coverage and volumes in the water area of the Sea of Japan.

clear days is larger than in other days. Since wind speeds at the stage of ice formation are greater than those at the stage of ice cover melting, the rate of autumn hummocking $\nu^{(A)}$ exceeds the hummocking rate $\nu^{(S)}$ in the spring. The plots of ice coverage and total ice volumes calculated by this model for the water area of the Sea of Japan are given in Fig. 5. The distributions of IC characteristics for open-sea areas and fast ice are shown by separate curves. The comparison of the curves of fast ice coverage $L_F^{(M)}$ and open-sea areas $L_S^{(M)}$ shows that in the period of IC formation in autumn (from the 31st to the 36th ten-year period), the fast ice areas are greater than those of open-sea ice. This is due to the temperature T_F^* being greater than the temperature T^*. The quantitative relationship between the integral characteristics of IC in the Sea of Japan is described by the regressions

$$L_S^{(M)} = c_L^{(1)} + c_L^{(2)} L_F^{(M)},$$
$$V_S^{(M)} = c_V^{(1)} + c_V^{(2)} V_F^{(M)},$$

where $L_S^{(M)}$, $L_F^{(M)}$ are the ice coverage values of the open sea and fast ice, and $V_S^{(M)}$, $V_F^{(M)}$ are the respective total volumes of ice; $c_L^{(1)}$, $c_L^{(2)}$, $c_V^{(1)}$, and $c_V^{(2)}$ are coefficients of regression lines. Here the characteristics of open-sea IC are dependent variables, and the independent variables are characteristics of fast ice: $c_L^{(1)}$ = –0.51138 ± 0.10505, $c_L^{(2)}$ = 1.39653 ± 0.13634, $c_V^{(1)}$ = (–2.45990 ± 0.65276)10^9, $c_V^{(2)}$ = 1.25366 ± 0.03028. The values of the determination coefficients of these regression lines equal to 0.975 and 0.968 demonstrate the high degree of linear correlation between the measured values and the values of dependent variables calculated by (1). The confidence domains of parameter values demonstrate their statistically significant difference from zero. The free terms $c_L^{(1)}$ 1 and $c_V^{(1)}$ in these equations are negative, thus justifying the following assumption: on the scale of the sea, the appearance of ice in the open sea is preceded by the appearance of fast ice. Indeed, the values of ice coverage and ice volumes in the open sea become positive only after these characteristics of fast ice have exceeded certain values.

CONCLUSIONS

The obtained results are based on the fact that the models describing the thermal evolution of IC can be formalized on the basis of the notions of the resource–consumer system, where the rate of interaction is characterized by the air temperature. The observed peculiarities of sea ice evolution can be described within the framework of these models.

The model of formation and destruction of fast ice is based on its being considered as a single floe, whose evolution is controlled by the temperature of the atmosphere. During spring melting, fast ice breaks up into separate fragments in accordance with laws of its fracturing. If the process is considered in terms of ice areas and thicknesses, the fast ice is schematized as an assemblage of floes with different thicknesses, interacting with one another. The melting of fast ice is described with allowance made for the division of areas into parts, which pass into the category of open-sea ice. A model was developed to describe the dynamics of the melt water that forms within the IC mass. The results of numerical modeling of ice cover evolution justify the assumptions made during the development of the models.

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