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HYDROPHYSICAL PROCESSES

Statistical Analysis of Ice Cover State Parameters in the Sea of Japan and Mathematical Modeling of Its Evolution

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Abstract—Data of a sample from long-term observations (made with an interval of ten days) of ice cover state parameters in the Sea of Japan are analyzed. The results of analysis are used to formulate a model describing the evolution of ice floe distribution in terms of area and thickness. The obtained model is used to construct a model describing the evolution of ice thickness. Particular cases are studied analytically. A method for parametric identification of the model is considered and its adequacy is assessed against observed distributions. The model can be used to predict the state of the ice cover in the Sea of Japan.

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INTRODUCTION

Studying multicomponent systems in various fields is based on the analysis and assessment of a large number of characteristics with diverse nature. The adequate quantitative description of such systems or a set of their elements, interacting with one another in a certain manner, is possible only if the number of available observational units (sample points or objects under observation) is sufficiently large and reflects the main features of the system: the presence of deterministic or stochastic properties, evolution and jump-like phases determined by changes in the functional structure of the system.

In this case, the system under study is the ice cover of the Sea of Japan, which is represented by a statistically reliable sample of long-term ten-day observations of the state parameters of ice cover (the material was presented by V.V. Plotnikov, Pacific Oceanological Institute, FED RAS). The external action was determined by the temperature and wind speed distributions at atmospheric horizons standard for such studies (material for 1960–2001 was presented by V.P. Tunegolovets, Pacific Oceanological Institute, FED RAS). The need for statistical studying is due to its ability to reveal the functional structure of the process of evolution and to identify the regional specific features of the Sea of Japan's ice cover. Moreover, the analysis of experimental data and the identification of the major properties of system functioning are prerequisites to the construction of a mathematical model of the phenomenon considered. Experimental data are also used for parametric identification of the model and the assessment of the adequacy of description of the process examined.

THE DISTRIBUTION OF ICE COVER STATE PARAMETERS

The parameters of ice cover state commonly used in its studies are ten-day values of its consolidation ratio S, ice cover thickness h, and the predominant size of floes F in each region of the sea with relatively uniform ice conditions [9, 16]. The consolidation ratio S (the ratio of ice cover area in a region to the region's area) is a dimensionless variable; h and F are measured in meters. The statistical analysis is based on a long-term observational series of 1961–1989, which characterizes the values of parameters in points in the ordinal measurement scales [6] (the boundaries of the appropriate intervals are specified by fixed sets of numbers $\{S_i^{(T)}: S_i^{(T)} = 1/10\}_{i=0–10}$, $\{h_i^{(T)}\}_{i=0–6}$, $\{F_i^{(T)}\}_{i=0–7}$, and $S_0^{(T)} = h_0^{(T)} = F_0^{(T)} = 0$). This period is described by an exhaustive and uniform series of independent observations based on the results of regular aerial reconnaissance and characterizing the state of the ice cover.

The elementary statistics for the dth ten-day period of the rth region was made by using the method of formation of original observations: digitized point value of each parameter within a certain interval (denoted by the symbol of the parameter being measured) is a uniformly distributed random variable. The estimates are supposed to be statistically consistent and continuous as functions of random arguments [10]. These properties enable linearization of estimates in a vicinity of the expectation of arguments. In this case, the sample mean and sample variance of a function $X = \varphi(x_1, x_2, E, x_n)$ is determined by the relationships

$$\bar{X} \approx \varphi(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n) \text{ and } \sigma^2(X) \approx \sum_{k=1}^{n} (\partial\varphi/\partial x_k)^2 \sigma^2(x_k) + \sum_{k=1}^{n} \sum_{l=k+1}^{n} (\partial\varphi/\partial x_k)(\partial\varphi/\partial x_l)\mathrm{cov}(x_k, x_l),$$

where $\mathrm{cov}(x_k, x_l)$ is the sample covariation of x_k and x_l. The mean and variance of random variable x_k uniformly distributed in the half-open interval $(x_{k-1}^{(T)}, x_k^{(T)}]$ are known to be $M(x_k) = (x_{k-1}^{(T)} + x_k^{(T)})/2$ and $D(x_k) = (x_k^{(T)} - x_{k-1}^{(T)})^2/12$. The arguments for the assessment of sample ice consolidation are the set of $\{S_i\}_{i=1-10}$ and the set of $\{p_{dri}^{(S)}\}_{i=1-10}$ of the occurrence frequency of ice area with the ith point of consolidation. Now

$$\bar{S}_{dr} \approx \sum_{i=1}^{10} \bar{p}_{dri}^{(S)} M(S_i)$$

and

$$\sigma^2(S_{dr}) \approx \sum_{i=1}^{10} [M^2(S_i)\sigma^2(p_{dri}^{(S)}) + \bar{p}_{dri}^{(S)2} D(S_i)],$$

where $\bar{p}_{dri}^{(S)}$ is the sample estimate of the mean frequency of occurrence, the sample variance of which is $\sigma^2(p_{dri}^{(S)})$. Indeed, random variable $S_i (i = 1–10)$ does not depend on its occurrence. Therefore, $\mathrm{cov}(p_{dri}^{(S)}, S_i) = 0$. It appears reasonable to determine the mean thickness of ice cover in terms of the random variable $h_{dr} = \sum_{i=1}^{10} \sum_{j=1}^{6} p_{dri}^{(Sh)} S_i h_j / S_{dr}$, where $p_{dri}^{(Sh)}$ is the frequency of joint occurrence of ice area with the ith consolidation level and the jth thickness level, the sample mean of which is $\bar{p}_{dri}^{(Sh)}$ and the sample variance is $\sigma^2(p_{dri}^{(Sh)})$. In this case we take into account the thicknesses and volumes of ice

$$\bar{h}_{dr} \approx \sum_{i=1}^{10} M(S_i) \sum_{j=1}^{6} \bar{p}_{drij}^{(Sh)} M(h_j)/\bar{S}_{dr},$$

$$\sigma^2(h_{dr}) \approx \left\{ \sum_{i=1}^{10} [a_i^2 \sigma^2(p_{dri}^{(S)}) + b_i^2 D(S_i)] + \sum_{j=1}^{6} c_j^2 D(h_j) + \sum_{i=1}^{10} \sum_{j=1}^{6} [q_{ij}^2 \sigma^2(p_{drij}^{(Sh)}) + a_i q_{ij} \mathrm{cov}(p_{dri}^{(S)}, p_{drij}^{(Sh)})] \right\} / \bar{S}_{dr}^2,$$

where $a_i = -\bar{h}_{dr} M(S_i)$, $b_i = \sum_{j=1}^{6} p_{drij}^{(Sh)} M(h_j) - \bar{p}_{dri}^{(S)} \bar{h}_{dr}$, $c_j = \sum_{i=1}^{10} \bar{p}_{drij}^{(Sh)} M(S_i)$, and $a_{ij} = M(S_i)M(h_j)$. The expressions are written taking into account the independence of measurements of S and h. The sample means $\bar{F}_{dr}$ and the variances

$$\bar{F}_{dr} \approx \sum_{i=1}^{7} \bar{p}_{dri}^{(F)} M(F_i) \text{ and}$$

and

$$\sigma^2(F_{dr}) \approx \sum_{i=1}^{7} [M^2(F_i)\sigma^2(p_{dri}^{(F)}) + \bar{p}_{dri}^{(F)2} D(F_i)],$$

where $\bar{p}_{dri}^{(F)}$ is the sample estimate of the mean occurrence frequency of individual ice floes with the ith predominant ice floe size; its variance is $\sigma^2(p_{dri}^{(F)})$. The value of F is much greater than S and h; therefore, logarithmic scale is used for F in the analysis. Such transformations are justified by the fact that, in real situations, relationships are sought not only between the original characteristics, but also between some monotonic functions of them.

Variations in these geometric parameters of ice cover in the Sea of Japan for the observation period under study are given in Fig. 1, where observations in the (1) autumn, (2) winter, and (3) spring seasons are marked (figures denote the position of the mean parameter value for these seasons). The results of statistical analysis are as follows. The consolidation histogram in Fig. 1a corresponds to a U-shape distribution. The first mode corresponds to ice with a low consolidation (ice cover forms in autumn and decays in spring), and the second mode corresponds to the ice cover in the water areas where ice occupies 0.6 and more of the total area. The mode lies in the consolidation value of 0.8. It is worth mentioning that in [3, 8] this value is given as an upper estimate of consolidation for a non-zero value of the analogue to hydrostatic pressure. Observations in this zone correspond to ice fields of large length and mature fast ice. This can be readily explained: there are regions in the sea where ice covers the major portion of the area and remains in this state until its thermal destruction in spring. This fact determines a feature of the mathematical model: it should allow for the situation in which a farther drop in the temperature at the maximum ice consolidation does not affect the latter or its complete derivative in this case is equal to zero.

The histogram of ice thickness in Fig. 1b corresponds to a bimodal distribution. According to Fig. 1e, the first mode corresponds to ice of the autumn and winter phases, and the second mode, to a mature state and the beginning of the spring phase. This can be attributed to the fact that, in the process of ice cover thermal destruction, the relative rate of changes in its area is much greater than the respective value for the thickness. This follows from the part of Fig. 1g (the center of the group of points 3) where lower values of consolidation correspond to higher values of thickness.

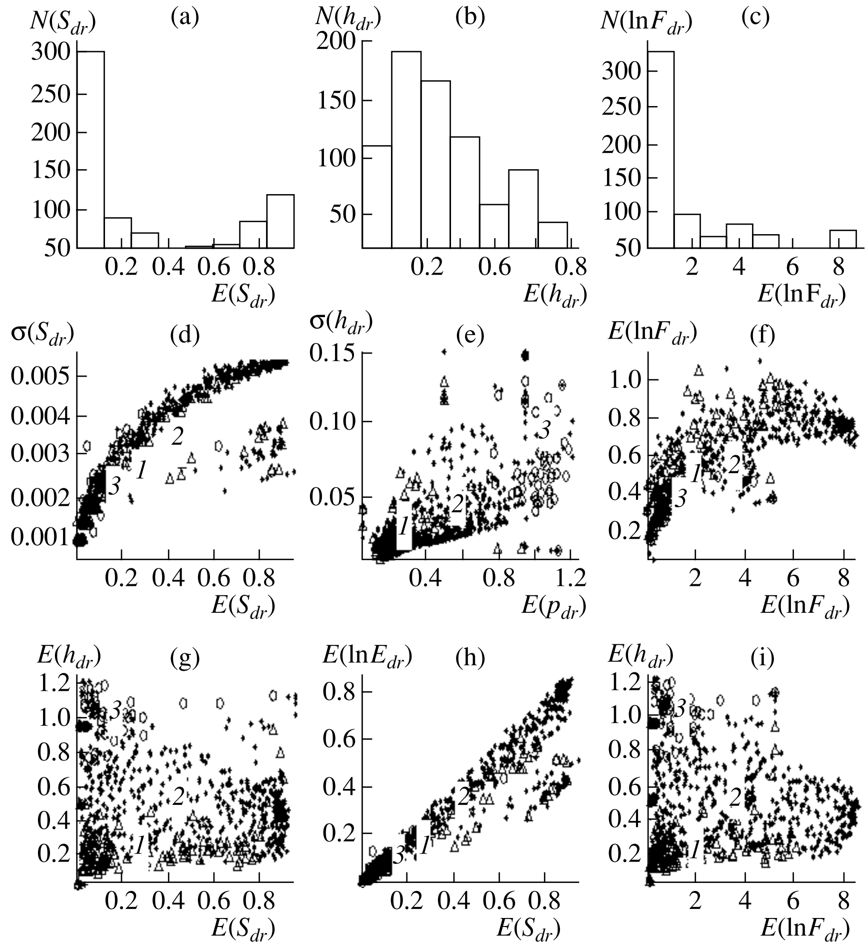

Fig. 1. Histograms of the distributions of (a–c) *S*, *h,* and ln(*F*); (d–f) joint distributions of sample means and their RMSD; (g–i) pair distributions of sample means of parameters.

The histogram of the logarithm of the size of individual floes in Fig. 1c is similar to the histogram in Fig. 1a: the sample correlation coefficient between S_{dr} and $\ln F_{dr}$ is equal to 0.946, that is, the size of individual floes changes in a regular manner with changes in consolidation.

The distribution of sample points in Fig. 1d is evident: the closer the consolidation to 1, the less the scatter of its values. The root-mean-square deviation (RMSD) σ of the characteristic is used as a measure of its scatter. The points concentrated in the right-hand part of Fig. 1d represent mostly the winter phase of evolution because RMSD of consolidation drops to zero toward the end of ice cover formation and the attainment of the mature state of ice. The total changes in the consolidation over the entire cycle of evolution somewhat meats the requirements to a reversible process, when ice formation in autumn can be described in terms of its destruction in spring and vice versa. The small difference between the autumn and spring phases is due to the fact that ice melting in spring is accompanied by its crushing, which facilitates and accelerates its destruction. It should be mentioned that the variations in S_{dr} are narrow, which corresponds to a narrow range of variations in $\sigma(S_{dr})$.

The distribution of sample points in Fig. 1e demonstrates a significant difference between the autumn and spring phases of ice cover evolution. During ice cover formation in autumn, $\sigma(h_{dr})$ first changes almost linearly and next stabilizes at a certain level. In contrast to the previous case, no fall in $\sigma(h_{dr})$ at large values of its argument is observed here. This is due to the continuous change in the thickness and the heterogeneous structure of the cover. A distinctive feature of the destruction of ice cover in spring is a decrease in $\sigma(h_{dr})$ with ice thickness remaining almost unchanged. This effect is due to ice thermal conductance and the deep penetration of ice by short-wave radiation [1]. The increase in air temperature is accompanied by ice cover melting both from inside and from both its surfaces. The increase in ice-free water area, the reflectance of which is much lower than that of ice, facilitates intense heating of water mass. Therefore, thin ice disappears first, whereas sufficiently thick ice floes do not change significantly. The area of the cover decreases, as does its heterogeneity $\sigma(h_{dr})$. The difference between the autumn and spring phases is also due to the different physicochemical properties of sea water. By the moment of ice appearance in autumn, a quasi-homogeneous freshened layer forms in the upper part of the water mass. Its formation is caused by sinking cooled denser seawater masses and rising warmer and lighter deep-water masses. The cooler waters have a higher salinity, and the effect of wind increases the intensity of mixing [13]. Therefore, the initial evolution takes place in a homogeneous medium. Contrary to that, the destruction of ice cover takes place in a heterogeneous medium.

The distribution of sample points in Fig. 1f suggests that the dynamics of $\ln F_{dr}$ is identical to that of S_{dr}. The only differences are the slope of the enveloping curve and the presence of an interval with zero values of RMSD. The first part of this statement is due to the difference between the specific rates of growth/drop of the values of these variables. By contrast with ice floe sizes, ice consolidation characterizes an inertial state of ice cover in the entire region; therefore, its dynamics is smooth. The other part of the observed effect can be accounted for as follows: large ice floes (with a size of more than 10^4 m) are present in the coastal zone and represent fast ice, which in spring and autumn is tolerant to changes in its size. The zero root-mean-square deviations indicate to this situation. The jump in $\sigma(\ln F_{dr})$ corresponds to the formation, as well as melting and destruction of large fast ice floes.

The pair distributions of parameters are given in Figs. 1g–1i. The layout of sample points in the cases g–i suggests the absence of significant correlations between S_{dr} and h_{dr} as well as between $\ln F_{dr}$ and h_{dr}. The scatter of S_{dr} for thin ice was found to be wide, which may be attributed to the presence of an inflection point in the thickness curve. Indeed, with a drop in temperature in the water area, a thin ice film forms first. And only after that, a notable increase in h can be seen. The case h suggests a one-to-one correspondence between S_{dr} and F_{dr}, which is plotted in a logarithmic scale (the coefficient of correlation is 0.946). Therefore, we can assume that within the framework of the accepted spatial discreteness, the areas of ice floes lost in the process of hummocking is insignificant. Indeed, if the inverse statement were true, it would have applied to the predominant size of ice floes as well. However, this is not the case.

Analysis of data on air temperature distributions shows that the temperature at the 2-m horizon, which is a standard horizon for ice cover studies, in the beginning of ice cover formation and that in the beginning of its melting are the same. Thus, the mean air temperature in the ten-day period before the first appearance of ice in the Sea of Japan's water area was $T_0 = -(8.4 \pm 4.2)$°C, whereas that in the moment of first loss of consolidation $T_{S1} = -(9.1 \pm 4.7)$°C. If we assume the moment of beginning of ice cover melting to coincide with the ten-day period when ice thickness passed through its maximum value, then $T_{h1} = -(7.8 \pm 4.3)$°C (to distinguish between these variables, we use different subscripts). Student's test shows no statistically significant difference between these values. Thus, analysis of the distributions and consolidation and thickness of ice cover in the Sea of Japan for ten-day periods shows that the assumption that the air temperature in the moments of primary ice cover formation and its primary destruction is valid.

KINETIC MODEL OF ICE COVER EVOLUTION

Ice cover on seas is commonly studied in terms of its consolidation and thickness. If the mass of ice formations is used as a dynamic variable of the system, then their area is to be determined for the calculation of the drift velocity (ice drift is controlled by the shear stresses at both sides of such formations). Here, independent dynamic variable of the system are the thickness h and area a of an individual ice floe. Their independence follows from Fig. 1g. Therefore, hereafter it is assumed that in any time moment, the set of ice floes can be arranged in accordance with the two-dimensional table of the areas and thicknesses of individual floes that has been compiled based on the original sample. In accordance with the results obtained in the previous section and simple qualitative reasoning, the evolution of ice cover is controlled by the combination of ice drift; direct formation of isolated primary ice floes; successive growth in the area of individual floes; the formation of the areas of the kth size as a result of aggregation of areas of the mth and jth size $a_k = a_m + a_j$, where $m + j = k$; removal of a number of floes from the kth size class because of their aggregation with other ice floes of arbitrary sizes.

The time step in the determination of the air temperature and wind speed used in this study is one day. Therefore, the time step considered in the analysis that follows is also equal to one day. In this case, the analysis of the respective Euler equations for the drift velocity shows that, for this time step, it is quasi-stationary and can be determined from simple relationships [5, 12]. Mass ice drift is supposed to take place in this case.

The reliability of the assumption regarding the character of aggregation of individual floes follows from the fact that the losses of their areas during collisions are insignificant (this follows from Fig. 1h). Now the area of an ice floe is an additive parameter of the evolution of ice cover: aggregation of ice floes results in the formation of a floe with an area equal to the sum of the floe areas. Direct observations show that two ice floes are commonly involved in collisions. The collisions of a larger number of ice floes are extremely rare; therefore, we will consider only the case of pair-wise aggregation of ice floes.

With the assumptions made, the dynamics of the density of ice floe number $n_{ah} = n(x, y, t, a, h)$ in open areas of the sea is determined by the result of integration of Boltzmann equation with respect to the momentums of individual ice floes and the provisions of Smolukhovskii equation describing the character of pair-wise collisions

$$\begin{aligned}\partial n_{ah}/\partial t + \partial u_i n_{ah}/\partial x_i + \partial \dot{h} n_{ah}/\partial h + \partial \dot{a} n_{ah}/\partial a \\ = f_{ah} + D\partial^2 n_{ah}/\partial h^2 + Q_{ah} + R_{ah},\end{aligned} \tag{1}$$

where $u = (u_1, u_2)$ is the ice drift velocity; $x = (x_1, x_2)$ are spatial coordinates; $\dot{h} \equiv dh/dt$ is the rate of the thermal increase in the thickness; $\dot{a} \equiv da/dt$ is the rate of the thermal increase in the area of an individual ice floe; $f_{ah} \equiv f(a, h_1, T)$ is the rate of formation/elimination of isolated ice floes with an initial gradation of the thickness and area a; D is diffusion constant (to simplify the reasoning, we assume $Q_{ah} \equiv Q(x, y, a, h, T)$, $R_{ah} \equiv R(x, y, a, h, T)$ are terms statistically describing the dynamics of aggregation and crushing of particles–floes [2, 4]. It is additionally assumed in (1) that the leveling of floe thicknesses in the ice cover takes place in isolated regions and that this process can be qualitatively described by diffusion.

It appears reasonable to perform the parameterization of $\dot{h}$ and $\dot{a}$ based on the assumption that the sea area containing the ice cover is limited in the space [14, 15, 19, 20]. It is obvious that $\dot{h}$ is proportional both to the current value of h and the current value of the resource $h_{\max} - h$ available for the thickness; here, $h_{\max}$ is the maximum thickness over the long-term observational period. Indeed, at the phase of the primary formation of ice cover, when h is small, the growth rate is a linear function of h. As the ice cover approaches its mature state $h \sim h_{\max}$, when the entire resource $h_{\max} - h$ is exhausted, the thickness stabilizes; therefore, at the phase of mature state, $\dot{h} = 0$. However, the situation during the phase of melting and mechanical destruction off ice is inverse. At the moment of final destruction of ice, this function becomes zero. The rates of processes is controlled by air temperature, ice cover thickness h_s, and a number of accompanying factors $F = (F_1, F_2, E, F_k)$ (the albedo of snow surface etc.). The results of statistical analysis of air temperature distribution show that the temperatures of the primary formation and primary melting of ice cover (hereafter, denoted by T^*) coincide; therefore

$$\dot{h} = f_h(T, T^*, h_S, F)(h_{\max} - h)h.$$

Function $f_h(T, T^*, h_s, F)$ can be approximated by its linear representation $\alpha(T - T^*) + \beta h_S + \sum_{l=1}^{k} \gamma_l (F_l - F_l^*)$, where F_l^* is the value of external factors at which the primary formation of ice cover begins. The values of estimates of the respective coefficients can be found from long-term observational series. However, such observations for ice cover in the Sea of Japan are fragmentary or absent. Therefore, we will consider a particular case when the environmental effect on the thickness of ice cover is characterized by air temperature and the thickness of ice cover is not taken into account, whereas the values of other external dynamic variables are fixed. Therefore, the thermal evolution of the thickness of ice cover can be written as

$$\begin{aligned}\dot{h} &= (T - T^*)[\alpha_h \Theta_1(T) + \alpha'_h \Theta_2(T)](h_{\max} - h)h, \\ \Theta_1(T) &= \Theta(T^* - T), \\ \Theta_2(T) &= \Theta(T - T^*),\end{aligned} \tag{2}$$

where $\Theta(z)$ is Heaviside function equal to 1 at $z > 0$ and 0 otherwise; α_h, α'_h are nonnegative proportionality factors. Their dimensions are (m °C day)$^{-1}$, that is, the numerical value of each coefficient characterizes the daily relative change in the thickness of the ice cover when air temperature changes by 1°C.

The change of sign of $\dot{h}$ takes place in the moment when the temperature of the air layer exceeds T^*. In the present case, T^* characterizes the combination of the ambient conditions at which primary ice formation begins. It is also apparent that its values for fast ice T_B^* and ice in the open part of sea regions T^* are different. Indeed, waters of the coastal zone are shallow and freshened by river waters and industrial effluents. Therefore, ice formation in open parts of the sea begins at lower air temperatures than in coastal areas. The thermal destruction of ice in open areas also begins earlier and at lower temperature because of the combined effect of solar radiation and warm currents of the Sea of Japan [18]. The numerical values of estimates of α_h, α'_h, T^*, and T_B^* are determined during a series of numerical experiments based on a sample of air temperature and ice thickness distributions.

If T is a periodic function, the result of its integration will be shifted in time with respect to the periodicity of T. Therefore, there exists a time lag between the minimum air temperature $T_{\min}$ and the maximum ice thickness. This regularity was detected in numerous studies, and in the case of ice cover in the Sea of Japan, it was confirmed, in particular, by the results of statistical analysis of the sample. If the rate of air cooling for a water body satisfies the condition $\dot{T}(t_{Gh})/[T^* - T(t_{Gh})]^2 < \alpha_h h_{\max}$, the curve of thickness has an inflection point $h(t_{Gh}) = \{h_{\max} - \alpha_h^{-1}\dot{T}(t_{Gh})/[T^* - T(t_{Gh})^2]\}/2$ at $t = t_{Gh}$. When the inequality is not satisfied, the inflection point in the thickness curve disappears and $h(t)$ becomes a step curve, i.e., rapid freezing up of small water bodies takes place. In other cases, there exists an "incubation period" t_{Gh} during which a drop in air temperature is accompanied by the primary formation of ice cover. In reality, the presence of this period is due to the following: at the preliminary phase of ice cover evolution, individual accumulations of ice germ appear; their aggregation and conglutination take place; the water area becomes covered by a thin ice film, and its thickness virtually does not change; after that the thickness of ice increases to its maximum.

During the thermal evolution of the area of an individual ice floe, its current resource is determined by the area $A^* - \int_0^{h_{\max}}\int_0^{A^*} n(a, h) a da dh \equiv A^* - A_\Sigma$, where A^* is the area of the region (the reliability of this is substantiated in [17]). In accordance with (2), we have for $\dot{a}$

$$\dot{a} = (T - T^*)[\alpha_a \Theta_1(T) + \alpha'_a \Theta_2(T)](A^* - A_\Sigma)a. \quad (3)$$

It appears reasonable to define f_{ah} as a rapidly decreasing function of a, for example, the hyperbola $C_f(T, h)(a_1 + a)^{-k}$, where the first factor is nonzero only for $h = h_1$ and $k > 2$ is a number. Function $C_f(T, h)$ is determined from the condition that the density of the number of ice floes that form during ice formation or that of the ice floes that have melted is determined by the expression $I = \int_0^{h_{\max}}\int_0^{A^*} f_{ah} da dh$. In the first case, it appears reasonable to assume that I is proportional to the water area $A^* - A_\Sigma$, and in the second case, proportional to the current area of thin ice A_1. In both cases, the rate of processes is controlled by the cooling $T^* - T$ of the near-ice air layer. Therefore, $I = (T^* - T)[C(A^* - A)\Theta_1(T) + C'A_1\Theta_2(T)]$, where C and C' are nonnegative factors. Because of $a_1 \ll A^*$, we have

$$\begin{aligned} C_f(T, h) &\approx (T^* - T)[C(A^* - A_\Sigma)\Theta_1(T) \\ &\quad + C'A_1\Theta_2(T)](k-1)a_1^{k-1}\delta_{h, h1}, \end{aligned} \quad (4)$$

where $\delta_{h, h1}$ is Kronecker delta. Equation (4) is written based on the assumption that in any moment, the set of individual ice formations can be uniquely arranged in accordance with the two-dimensional table $\Omega = \{(a, h): 0 \le a \le A^*, 0 \le h \le h_{\max}\}$ containing the areas and thicknesses. Under condition that the area of the result of aggregation does not exceed the area of the region, this result is an element of the table. Therefore, in this case, the so-called coagulation term [4] takes the form

$$Q_{ah} = \frac{1}{2}\iint_{\Omega_{ah}} \beta(ah - a'h', a'h') n_{a-a', (ah - a'h')/(a-a')} n_{a', h'} \times da'dh' - n_{ah}\iint_{\Omega} \beta(ah, a'h') n_{a', h'} da'dh',$$

where $\Omega_{ah} = \{(a', h'): 0 \le a' \le a, 0 \le h' \le h\}$; $\beta(z, y)$ is the kernel of kinetic equation, which is a symmetric function. It was written taking into account the fact that aggregation is commonly described in terms of volumes or masses, rather than areas of individual ice floes. According to the first term, the proposed mechanism of aggregation is as follows: the area of the result of aggregation is equal to the sum of areas of ice floes and the thickness is calculated as the sum of volumes of original ice floes divided by their total area.

The result of ice crushing is supposed to have the thickness of the original ice floe. Modification of R_{ah} in Melzak kinetic equation [2] takes the form

$$R_{ah} = \int_a^{A^*} \gamma(a', a, h) n_{a', h} da' - a^{-1} n_{a, h} \int_0^a \gamma(a, a', h) da',$$

where $\gamma(a', a, h)$ is the probability that ice floes with an area of a will form when ice floes with an area of $a' > a$ crush. Function $\gamma(a', a, h)$ is normalized so that integral $P_{ah} = a^{-1}\int_0^a \gamma(a, a', h) da'$ is equal to the probability that an ice floe with an area of a will be destroyed within a unit time.

Inasmuch as the complete cycle of ice cover evolution is considered, the initial distribution is taken equal to zero. The boundary conditions readily follow from (2) and (3): the absence of fluxes of the number of ice floes at the extreme boundaries of gradations of ice floe sizes and thicknesses. Thus, the cycle of ice cover evolution is considered within a system closed with respect to fluxes of matter.

THE DISTRIBUTION OF ICE COVER THICKNESS

The distribution of ice thickness $A_h \equiv A(x, y, t, h)$ for open sea areas is obtained by multiplying (1) by a with a subsequent integration with respect to a

$$\begin{aligned} &\partial A_h/\partial t + \partial u_i A_h/\partial x_i + \partial \dot{h} A_h/\partial h \\ &\quad = (T^* - T) f_{Ah} + D\partial^2 A_h/\partial h^2, \\ &f_{Ah} = (\alpha_a A_h + \alpha_{ah}\delta_{h, h1})(A^* - A_\Sigma)\Theta_1(T) \\ &\quad + [\alpha'_a (A^* - A_\Sigma) A_h + \alpha'_{ah} A_1 \delta_{h, h1}]\Theta_2(T). \\ &A_\Sigma = \int_0^{h_{\max}} A_h dh. \end{aligned} \quad (5)$$

The initial and boundary conditions for (5) take the form

$$A_h(x, y, 0) = 0 \quad \text{at} \quad \dot{h} A_h|_{h = 0, h_{\max}} = 0. \quad (6)$$

In coastal zones, a distinction is made between the ice of their open parts A_h and the fast ice $A_h^{(B)}$, fragments of which that form during spring thermal destruction and mechanical crushing are carried out into the open sea and pass into the category of open-sea ice. To take this effect into consideration, a term should be introduced into the right-hand part of (5) to account for the increment of the area of open-sea ice of different thicknesses due to fast-ice fragments carried out into

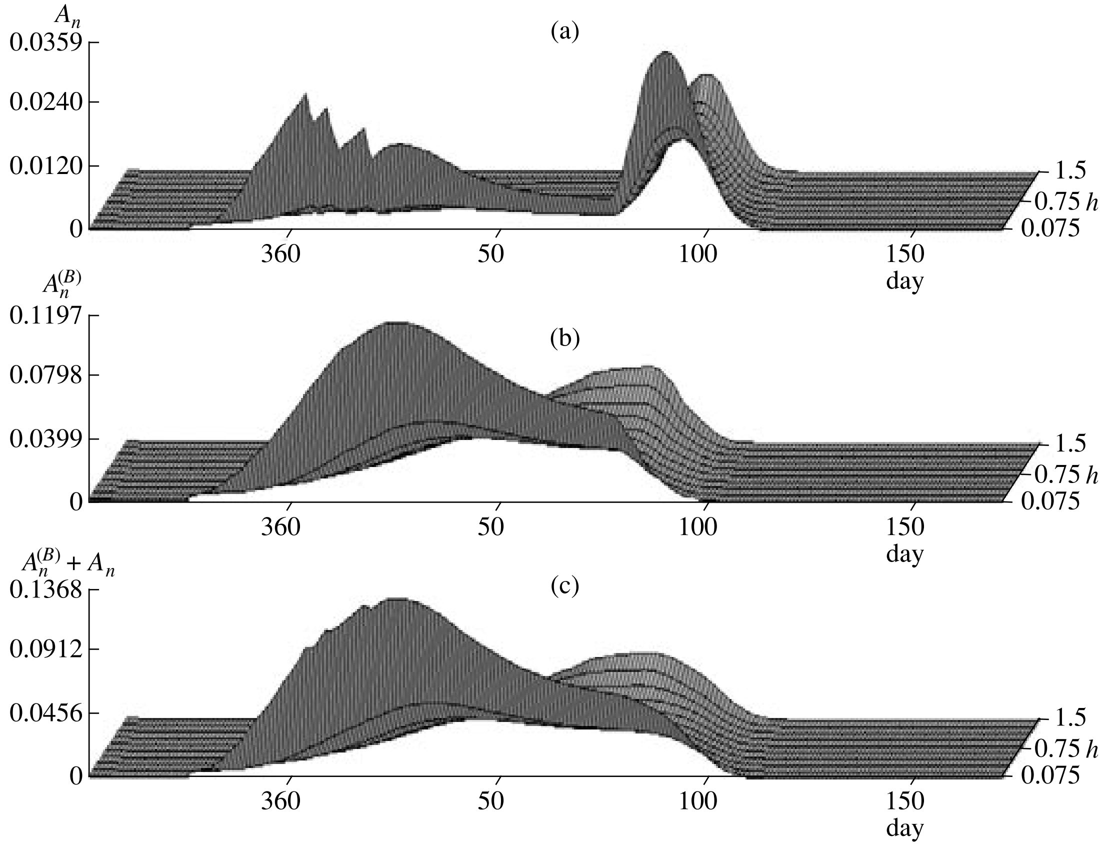

Fig. 2. Distributions of the thickness of ice cover in the water area of Peter the Great Bay ((a) open part, (b) fast ice, and (c) joint distribution).

the open sea. Accordingly, the fast-ice areas decrease by the same values. This term is proportional to the current area of fast ice $A_h^{(B)}$, that is, it can be written as $b_h(T)A_h^{(B)}$, where the proportionality factor depends on air temperature and ice thickness, because the crushed thin ice is finer than crushed thick ice. When formalizing these features of ice crushing, we can restrict ourselves by the linear representation

$$b_h(T) = [b_a(T - T_B^*) - b_h h]\Theta_2(T),$$

where b_a, b_h are nonnegative parameters of fast ice crushing. Thus, the dynamics of ice area in coastal regions can be written as

$$\begin{aligned} &\partial A_h/\partial t + \partial u_i A_h/\partial x_i + \partial \dot{h} A_h/\partial h \\ &= (T^* - T)f_{Ah} + D\partial^2 A_h/\partial h^2 + b_h(T)A_h^{(B)}, \\ &\partial A_h^{(B)}/\partial t + \partial \dot{h} A_h^{(B)}/\partial h = (T_B^* - T)f_{Ah} \\ &+ D\partial^2 A_h^{(B)}/\partial h^2 - b_h(T)A_h^{(B)}, \end{aligned} \qquad (7)$$

where A_Σ in the expression for f_{Ah} from (5) is determined by $A_\Sigma = \int_0^{h_{\max}}(A_h + A_h^{(B)})dh$. The assessment of the agreement between models (5) and (7) and field observations, as well as the calibration of model parameters are performed in a series of numerical experiments.

If, during its formation and in its mature state, fast ice occupies the major portion of the water area in the region (the relatively closed coastal zones and small lakes), its area at this phase is determined by the integral with respect to h of the second equation (7)

$$A_\Sigma^{(B)}(t) = A^*\left\{1 - \frac{\alpha_a A^* + \alpha_{ah}h_1}{\alpha_a A^* + \alpha_{ah}h_1 \exp[(\alpha_a A^* + \alpha_{ah}h_1)(T_B^* - \bar{T})t]}\right\},$$

where $\bar{T} = \frac{1}{t}\int_0^t T\,dt$ is the current air temperature in the coastal zone of the water area. Curve $A_\Sigma^{(B)}(t)$ and the thickness have an inflection point $A_\Sigma^{(B)}(t_{GA}) = \{A^* - \alpha_a^{-1}\alpha_{ah}h_1 - \alpha_a^{-1}\dot{T}(t_{GA})/[T_B^* - T(t_{GA})]^2\}/2$ at $\dot{T}(t_{GA})/[T_B^* - T(t_{GA})]^2 < \alpha_a A^* + \alpha_{ah}h_1$. When the inequality is not true, the inflection point $A_\Sigma^{(B)}$ disappears, which corresponds to a rapid development of a thin ice film in the coastal water area. When the inequality is true, there exists a

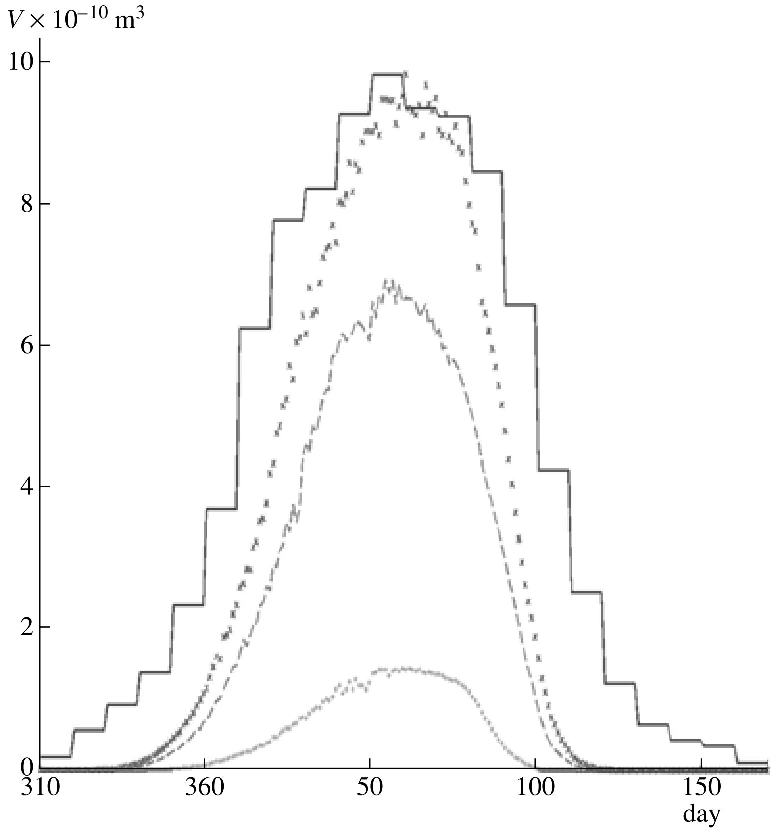

Fig. 3. Dynamics of ice volumes in the Sea of Japan at changes in air temperature. The solid line is the rate of variations in ice volume, criss-crosses denote model calculations, dashed line is for variations in ice volume at a 1°C temperature increase, dotted line is the same for an increase by 5°C.

period t_{GA} within which a primary ice cover forms at an decrease in air temperature.

Details of the evolution of ice cover in individual parts of the sea can be derived from the analysis of changes in A_h and $A_h^{(B)}$ in these regions. In particular, studying the distributions in Fig. 2 (the time coordinate is the current day of the year, and the respective variables are normalized by the area of the open sea region) enables the assessment of the rate of fast ice destruction in Peter the Great Bay. Analysis shows that during ice cover melting in the spring, the ice area in open zones in the bay increases. This increase is due to the fragments of fast ice carried here. These facts are in good agreement with field observations [9, 18].

Model (7) reproduces the real processes of fast ice destruction; therefore, the proposed apparatus can be used for prediction, which is of great importance for the Sea of Japan, because breaking of fast ice is accompanied by its individual parts being carried into the open sea with numerous fishermen occupying them. The rescue problem appears in this case.

COMPUTATIONAL EXPERIMENTS

Parameter calibration and verification of the model were based on a series of computational experiments. Each experiment involved the search for the extremum of a functional that characterizes the discrepancy between the observed ten-day distributions of ice thickness and the respective distributions calculated from (5)–(7). The sample of ten-day distributions of ice thickness over the ice cover area values was formed based on the frequencies of joint occurrence of ice areas and thicknesses of different gradations. This was made with the use of the original set of long-term observations of ice consolidation and thickness.

Different methods were used to assess the agreement between the models (5) and (7) and field data. Specifically, the necessary parameters were estimated using one part of the sample (observations from 1961 to 1986), whereas the model verification was based on the other part of the sample (observations from 1987 to 1989). The agreement between the simulated and observed values of the total area of ice in the dth ten-day period of the Yth year was estimated using the relationship accepted in the studies of sea ice [7]

$$P_{d,Y} = \frac{m_{d,Y}}{n_{d,Y}}100\%,$$

where $m_{d,Y}$ is the number of correct predictions of the total area of ice; $n_{d,Y}$ is the total number of predictions that can be assessed with the available field data. The values of $P_{d,Y}$ are no less than 68%, thereby indicating that the model adequately describes the process considered.

The calculations show that the numerical estimates of parameters are determined by the intervals: $\hat{T}_B^* = -(6.3 \pm 0.8)$°C; $\hat{T}^* = -(7.6 \pm 0.9)$°C; $\hat{\alpha}_h = (2.842 \pm 0.209)10^{-3}$; $\hat{\alpha}'_h = (8.051 \pm 1.137)10^{-3}$; $\hat{\alpha}_h A^* = (2.486 \pm 0.175)10^{-2}$; $\hat{\alpha}'_a A^* = (5.980 \pm 0.186)10^{-2}$; $\alpha_{ah} A^* = (1.843 \pm 0.162)10^{-3}$; $\hat{\alpha}'_{ah} A^* = (1.473 \pm 0.125)10^{-3}$; $\hat{D} = (1.310 \pm 0.021)10^{-3}$; $\hat{b}_{ah}^{(T)} = (8.740 \pm 1.096)10^{-3}$; $\hat{b}_{ah}^{(h)} = (2.185 \pm 0.046)10^{-3}$. The dimensions of α_h and α'_h are (m °C day)$^{-1}$; to make the recording of parameter estimates more convenient, parameters α_a, α'_a, α_{ah}, α'_{ah} are given in the scale of the open area and their dimensions are (m^2 °C day)$^{-1}$; other dimensions are m^2/day for D, (°C day)$^{-1}$ for $b_{ah}^{(T)}$, and (m day)$^{-1}$ for $b_{ah}^{(h)}$. According to estimates, the rate of thermal destruction of ice cover $\hat{\alpha}'_h$ is almost three times as high as that of its formation $\hat{\alpha}_h$. This fact is in agreement with the results of statistical analysis.

Once the model is verified, it can be used to perform forecasting experiments aimed to study the effect of possible climate changes on the ice cover of the Sea of Japan. Given the present-day rate of anthropogenic CO_2 release into the atmosphere, the probability of such changes is appreciable. A very likely increase in the temperature is due to the existence of the so-called greenhouse effect of the Earth: short-wave solar radiation heats the Earth, whereas its radiation lies in a longer wave range. Therefore, an increase in CO_2 concentration significantly reduces the release of Earth's own radiation into the space and facilitates the temperature increase. Thus, according to data of mathematical modeling of the atmosphere–ocean system [11], a twofold increase in CO_2 concentration in the atmosphere will cause its mean temperature to rise from –19.2°C to –17.54°C. At the same time the precipitation will increase from 2.04 to 2.15 mm/day. The interest to this problem can be attributed to the need for quantitative estimates of the environmental impact of industrial enterprises to perform environmental expert appraisals. Moreover, such forecasts are of use for the comprehensive assessment of the possible state of climate system.

The calculations were made at the same wind regime in the 2-m above-ice water layer as that used for model verification. The vector of model parameters was represented by their estimates. Air temperature was increased with a step of 0.5°C. The results of calculations in the form of plots of total ice volumes are given in Fig. 3. The scale of measurements of ice volume is 10^{10} m^3. According to these results, as temperature increases by 1°C, the total losses of ice volume amount to 1.695×10^{12} m^3 (19% of the present-day level), and at an increase by 5°C, these losses are 6.092×10^{12} m^3 (69% of the present-day level). To more clearly represent these figure, let us express them in terms of the numbers of regions of the covered sea at a 1-m ice thickness. In the first case, the total losses are equivalent to the volume of ice within 249 such regions, and in the second case, to the volume of 826 regions.

These results give a rough estimate of the possible consequences of climate changes, because global warming will cause an increase in the amount of precipitation and affect the salt regime of the sea, which will have its effect on the beginning of ice cover formation.

CONCLUSIONS

The obtained results are based on the fact that, the process of seawater adaptation to the thermal and wind impact manifests itself in the formation of an intermediate medium (sea ice) the dynamics of state parameters of which is determined by a set of limiting parameters. In particular, an increase in the thickness of ice and snow reduces the heat flux through ice. Later, this flux reaches the level of ice flux from water into the ice. Brine (a liquid phase with a high salt content) migration from ice into water also causes a decrease in heat flux through ice, an increase in under-ice water salinity with an increase in its density. Therefore, this water will freeze up at lower temperature than that of the ice that has already formed. The increase in water salinity causes its active mixing, and cooled and more saline water is replaced by warmer waters from the depth, thereby increasing heat flux to the lower surface of ice. The combination of these factors limits the growth of ice thickness.

The constructed kinetic model of ice cover evolution is used as an intermediate link in the construction of the model of thickness distribution. However, its use for detailed studies of the evolution of ice cover of freezing seas also appears promising.

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